Topic 2: Comparing Numbers and Absolute Value

Rational numbers are numbers that can be expressed as one integer divided by another non-zero integer. Examples of rational numbers are $\frac{3}{4}$, $-\frac{7}{8}, \frac{3}{-1}$, 0.75, and $\sqrt{9}$.

Irrational numbers are numbers that cannot be expressed as one integer divided by another integer. Examples of irrational numbers are $\sqrt{2}, -\sqrt{3}$, π and 0.474774777....

The **absolute value** of a number a, represented as $|a|$, is the distance between the number a and zero. Because distance is a measurement, the absolute value of a number is always positive.
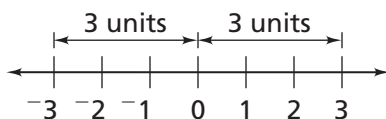
Opposites, like –3 and 3, have the same absolute value, 3, because they are each three units from zero.

Problem 2.1

Alexis, Brandon, Jacob, and Madison are playing a game. Each player gets a card with a number on it. The reverse side of the card contains a hidden letter. The four players line up from least to greatest. If they are correct, the hidden letters spell a word.

A. 1. For Round 1, Alexis has 0, Brandon has –2, Jacob has 2, and Madison has –1. How should the students line up?

2. When they reveal their letters, they spell the word NICE. Assign each letter to the proper student.

3. Which students have numbers that have the same absolute value?

B. 1. For Round 2, Alexis has $\frac{2}{3}$, Brandon has $-\frac{3}{5}$, Jacob has $-\frac{1}{2}$, and Madison has $\frac{1}{10}$. How should the students line up?

2. When they reveal their letters, they spell AMTH. What mistake do you think they made?

C. For Round 3, Alexis has $\sqrt{7}$, Brandon has $\sqrt{9}$, Jacob has 4, and Madison has 2. Alexis is not quite sure where to stand. Madison tells her that $\sqrt{4}$ is 2 and $\sqrt{9}$ is 3. Where should Alexis stand?

D. The students decide to make another set of cards, $-0.3\overline{3}$, $-\pi$, $-\sqrt{2}$ and $-\frac{1}{3}$.

    1. List the numbers from least to greatest

    2. Could OOPS be the hidden word?

Exercises

Use the following list of numbers for Exercise 1–5.

$-\frac{1}{2}$	$-2\frac{1}{2}$	π	-0.5	$\sqrt{4}$
4	$\frac{1}{4}$	$\sqrt{12}$	-2	0.25
0	$\sqrt{3}$	$-\sqrt{4}$	$\frac{1}{2}$	2
$-0.3\overline{3}$	-4	$-\frac{3}{5}$	3	2.5

1. List all of the rational numbers.

2. List all of the irrational numbers.

3. Give an approximate location for each number on a number line.

4. Which numbers have the same value?

5. Which numbers have the same absolute value?

6. a. Order the numbers $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$, and $\frac{1}{5}$, from least to greatest.

    b. As the denominator of a fraction increases, does the resulting positive fraction get larger or smaller?

    c. Does your rule apply for $-\frac{1}{2}, -\frac{1}{3}, -\frac{1}{4}, -\frac{1}{5}$? Explain.

Topic 2: Comparing Numbers and Absolute Value

Mathematical Goals

- Compare rational and irrational numbers by using the symbols $\leq, \geq, <, >$, and $=$.
- Apply the concept of absolute value.

Guided Instruction

Remind students that the prefix *ir* means "not," so *irrational* means "not rational."

Tell students that every rational number corresponds to a point on the number line. Draw a number line marked in units between –5 and 5. Remind students of the concept of absolute value.

- *Locate -3 on the number line.* (From zero, go 3 units to the left.)
- *Locate $-\frac{7}{8}$ on the number line.* (Between zero and -1, closer to -1; it is $\frac{7}{8}$ units to the left of zero.)
- *What are the absolute values of –3 and $-\frac{7}{8}$?* (3 and $\frac{7}{8}$, respectively)
- *Compare -3 and $-\frac{7}{8}$ using $<$.* ($-3 < -\frac{7}{8}$)
- *Locate $\sqrt{9}$ on the number line.* (Because $\sqrt{9} = 3$, go 3 units to the right.)
- *Is π a rational number?* (No) *Why not?* (It cannot be expressed as a ratio of two integers.)
- *Can π be located on a number line?* (Not exactly, only an approximation of π can be located as a point on the line.)
- *Give your best estimate as to where to locate π on the number line.* (π is approximately 3.14, so just to the right of 3.)
- *Compare $\sqrt{9}$ and π using $>$.* ($\pi > \sqrt{9}$)

When solving Problem 2.1, have four students act out the problem. Provide cards with additional numbers and letters to give students additional practice ordering rational and irrational numbers.

You will find additional work on comparing rational numbers in the grade 6 unit *Bits and Pieces I*.

ACE Assignment Guide for Topic 2

Core 1–6

Answers to Topic 2

Problem 2.1

A. 1. Brandon, Madison, Alexis, Jacob

   2. Brandon, N; Madison, I; Alexis, C; Jacob, E

   3. Brandon and Jacob

B. 1. Brandon, Jacob, Madison, Alexis

   2. Answers may vary. Sample: Brandon and Jacob got confused with the negative fractions. Since $\frac{3}{5} > \frac{1}{2}$, $-\frac{3}{5} < -\frac{1}{2}$ since it is further from zero.

C. Between 2 and 3, but closer to 3.

D. 1. $-\pi$, $-\sqrt{2}$, then $-\frac{1}{3}$ and -0.33 are equal.

   2. No. The word would end in a double letter.

Exercises

1. $-\frac{1}{2}$, $-2\frac{1}{2}$, -0.5, $\sqrt{4}$, 4, $\frac{1}{4}$, -2, 0.25, 0, $-\sqrt{4}$, $\frac{1}{2}$, 2, $-0.3\overline{3}$, -4, $-\frac{3}{5}$, 3, 2.5

2. π, $\sqrt{12}$, $\sqrt{3}$

3. See Figure 1.

4. $-\sqrt{4} = -2$; $-\frac{1}{2} = -0.5$; $\frac{1}{4} = 0.25$; $2 = \sqrt{4}$

5. $|-\sqrt{4}| = |-2| = |2| = |\sqrt{4}|$;

   $\left|-\frac{1}{2}\right| = |-0.5| = \left|\frac{1}{2}\right|$;

   $\left|\frac{1}{4}\right| = |0.25|$; $|-4| = |4|$; $\left|-2\frac{1}{2}\right| = |2.5|$

6. a. $\frac{1}{5}$, $\frac{1}{4}$, $\frac{1}{3}$, $\frac{1}{2}$

   b. The resulting fraction gets smaller.

   c. Answers may vary. Sample: No. For negative numbers, an increasing denominator results in larger fractions.

Figure 1